HYDRAULIC TURBINES

Introduction:
The device which converts hydraulic energy into mechanical energy or vice versa is known as Hydraulic Machines. The hydraulic machines which convert hydraulic energy into mechanical energy are known as Turbines and that convert mechanical energy into hydraulic energy is known as Pumps.

Fig. shows a general layout of a hydroelectric plant.
It consists of the following:

1. A **Dam** constructed across a river or a channel to store water. The reservoir is also known as **Headrace**.

2. Pipes of large diameter called **Penstocks** which carry water under pressure from storage reservoir to the turbines. These pipes are usually made of steel or reinforced concrete.

3. **Turbines** having different types of vanes or buckets or blades mounted on a wheel called runner.

4. **Tailrace** which is a channel carrying water away from the turbine after the water has worked on the turbines. The water surface in the tailrace is also referred to as tailrace.

**Important Terms:**

**Gross Head** \( (H_g) \): It is the vertical difference between headrace and tailrace.

**Net Head** \( (H) \): Net head or effective head is the actual head available at the inlet of the to work on the turbine.

\[
H = H_g - h_L
\]

Where \( h_L \) is the total head loss during the transit of water from the headrace to tailrace which is mainly head loss due to friction, and is given by

\[
h_f = \frac{4fLV^2}{2gd}
\]

Where \( f \) is the coefficient of friction of penstock depending on the type of material of penstock

\( L \) is the total length of penstock

\( V \) is the mean flow velocity of water through the penstock

\( D \) is the diameter of penstock and

\( g \) is the acceleration due to gravity
TYPES OF EFFICIENCIES

Depending on the considerations of input and output, the efficiencies can be classified as

(i) Hydraulic Efficiency

(ii) Mechanical Efficiency

(iii) Overall efficiency

(i) Hydraulic Efficiency: ($\eta_h$)

It is the ratio of the power developed by the runner of a turbine to the power supplied at the inlet of a turbine. Since the power supplied is hydraulic, and the probable loss is between the striking jet and vane it is rightly called hydraulic efficiency.

If R.P. is the Runner Power and W.P. is the Water Power

\[
\eta_h = \frac{R.P.}{W.P.} \quad (01)
\]

(ii) Mechanical Efficiency: ($\eta_m$)

It is the ratio of the power available at the shaft to the power developed by the runner of a turbine. This depends on the slips and other mechanical problems that will create a loss of energy between the runner in the annular area between the nozzle and spear, the amount of water reduces as the spear is pushed forward and vice-versa.

and shaft which is purely mechanical and hence mechanical efficiency.

If S.P. is the Shaft Power

\[
\eta_m = \frac{S.P.}{R.P.} \quad (02)
\]

(iii) Overall Efficiency: ($\eta$)

It is the ratio of the power available at the shaft to the power supplied at the inlet of a turbine. As this covers overall problems of losses in energy, it is known as overall efficiency. This depends on both the hydraulic losses and the slips and other mechanical problems.
that will create a loss of energy between the jet power supplied and the power generated at the shaft available for coupling of the generator.

\[ \eta = \frac{S.P.}{W.P.} \]

(03)

From Eqs 1, 2 and 3, we have

\[ \eta = \eta_h \times \eta_m \]

**Classification of Turbines**

The hydraulic turbines can be classified based on type of energy at the inlet, direction of flow through the vanes, head available at the inlet, discharge through the vanes and specific speed. They can be arranged as per the following table:

<table>
<thead>
<tr>
<th>Turbine Name</th>
<th>Type of energy</th>
<th>Head Discharge</th>
<th>Discharge</th>
<th>Direction of flow</th>
<th>Specific Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelton Wheel</td>
<td>Impulse, Kinetic</td>
<td>High</td>
<td>Low</td>
<td>Tangential to runner</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Head &gt; 250 m to 1000 m</td>
<td></td>
<td></td>
<td>&lt;35 Single jet</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Low</td>
<td></td>
<td>35 – 60 Multiple jet</td>
</tr>
<tr>
<td>Francis Turbine</td>
<td>Reaction Turbine</td>
<td>Medium</td>
<td>Medium</td>
<td>Radial flow Mixed Flow</td>
<td>Medium</td>
</tr>
<tr>
<td>Kaplan Turbine</td>
<td>Kinetic + Pressure</td>
<td>Low &lt; 30 m</td>
<td>High</td>
<td>Axial Flow</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>300 to 1000</td>
</tr>
</tbody>
</table>

As can be seen from the above table, any specific type can be explained by suitable construction of sentences by selecting the other items in the table along the row.
**PELTON WHEEL OR TURBINE**

Pelton wheel, named after an eminent engineer, is an impulse turbine wherein the flow is tangential to the runner and the available energy at the entrance is completely kinetic energy. Further, it is preferred at a very high head and low discharges with low specific speeds. The pressure available at the inlet and the outlet is atmospheric.

The main components of a Pelton turbine are:

(i) **Nozzle and flow regulating arrangement:**

Water is brought to the hydroelectric plant site through large penstocks at the end of which there will be a nozzle, which converts the pressure energy completely into kinetic energy. This will convert the liquid flow into a high-speed jet, which strikes the buckets or vanes mounted on the runner, which in-turn rotates the runner of the turbine. The amount of water striking the vanes is controlled by the forward and backward motion of the spear. As the water is flowing in the annular area between the annular area between the
nozzle opening and the spear, the flow gets reduced as the spear moves forward and vice-versa.

(ii) **Runner with buckets:**

*Runner* is a circular disk mounted on a shaft on the periphery of which a number of buckets are fixed equally spaced as shown in Fig. The buckets are made of cast-iron cast-steel, bronze or stainless steel depending upon the head at the inlet of the turbine. The water jet strikes the bucket on the splitter of the bucket and gets deflected through $(\alpha)\ 160-170^0$.

(iii) **Casing:**

It is made of cast-iron or fabricated steel plates. The main function of the casing is to prevent splashing of water and to discharge the water into tailrace.

(iv) **Breaking jet:**

Even after the amount of water striking the buckets is completely stopped, the runner goes on rotating for a very long time due to inertia. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of bucket with which the rotation of the runner is reversed. This jet is called as breaking jet.
From the impulse-momentum theorem, the force with which the jet strikes the bucket along the direction of vane is given by

\[ F_x = \text{rate of change of momentum of the jet along the direction of vane motion} \]

\[ F_x = \left(\text{Mass of water / second}\right) \times \text{change in velocity along the } x \text{ direction} \]
Work done per second by the jet on the vane is given by the product of Force exerted on the vane and the distance moved by the vane in one second
\[\text{W.D./S} = F_x \times u\]
\[= \rho a V_1 [V_{w1} + V_{w2}] u\]

Input to the jet per second = Kinetic energy of the jet per second
\[= \frac{1}{2} \rho a V_1^3\]

Efficiency of the jet = \[\frac{\text{Output/second}}{\text{Input/second}} = \frac{\text{Work done/second}}{\text{Input/second}}\]
\[\eta = \rho a V_1 [V_{w1} + V_{w2}] u \]
\[= \frac{1}{2} \rho a V_1^3\]
\[= \frac{2u [V_{w1} + V_{w2}]}{V_1^2}\]

From inlet velocity triangle, \(V_{w1} = V_1\)
Assuming no shock and ignoring frictional losses through the vane, we have \(V_{r1} = V_{r2} = (V_1 - u_1)\)

In case of Pelton wheel, the inlet and outlet are located at the same radial distance from the centre of runner and hence \(u_1 = u_2 = u\)

From outlet velocity triangle, we have \(V_{w2} = V_{r2} \cos \phi - u_2\)
\[= (V_1 - u) \cos \phi - u\]

\[F_x = \rho a V_1 [V_1 + (V_1 - u) \cos \phi - u]\]
\[F_x = \rho a V_1 (V_1 - u) [1 + \cos \phi]\]

Substituting these values in the above equation for efficiency, we have
\[\eta = \frac{2u [V_1 + (V_1 - u) \cos \phi - u]}{V_1^2}\]
\[\eta = \frac{2u [V_1 - u + (V_1 - u) \cos \phi]}{V_1^2}\]
The above equation gives the efficiency of the jet striking the vane in case of Pelton wheel.

To obtain the maximum efficiency for a given jet velocity and vane angle, from maxima-minima, we have

\[
\frac{d\eta}{du} = 0
\]

\[
\Rightarrow \frac{d\eta}{du} = \frac{2u}{V_i^2} (1 + \cos\phi) \frac{d}{du} (uV_i - u^2) = 0
\]

\[V_i - 2u = 0\]

or \[u = \frac{V_i}{2}\]

i.e. When the bucket speed is maintained at half the velocity of the jet, the efficiency of a Pelton wheel will be maximum. Substituting we get,

\[
\eta_{\text{max}} = \frac{2u}{(2u)^2} (2u - u)(1 + \cos\phi)
\]

\[
\eta_{\text{max}} = \frac{1}{2} [1 + \cos\phi]
\]

From the above it can be seen that more the value of \(\cos\phi\), more will be the efficiency. Form maximum efficiency, the value of \(\cos\phi\) should be 1 and the value of \(\phi\) should be 0°. This condition makes the jet to completely deviate by 180° and this, forces the jet striking the bucket to strike the successive bucket on the back of it acting like a breaking jet. Hence to avoid this situation, at least a small angle of \(\phi\) = 5° should be provided.
Dec-06/Jan07
6  a.i) Sketch the layout of a PELTON wheel turbine showing the details of nozzle, buckets and wheel when the turbine axis is horizontal (04)
   ii) Obtain an expression for maximum-efficiency of an impulse turbine. (06)

July 06
6  (a) With a neat sketch explain the layout of a hydro-electric plant (06)
   (b) With a neat sketch explain the parts of an Impulse turbine. (06)

Jan 06
6  (a) What Is specific speed of turbine and state Its significance. (04)
   (b)Draw a neat sketch of a hydroelectric plant and mention the function of each component. (08)

Jan 05
6  (a) Classify the turbines based on head, specific speed and hydraulic actions. Give examples for each. (06)
   (b) What is meant by Governing of turbines? Explain with a neat sketch the governing of an impulse turbine (06)

July 04
5  (a) Explain the classification of turbines. (08)
The head at the base of the nozzle of a Pelton wheel is 640 m. The outlet vane angle of the bucket is 15°. The relative velocity at the outlet is reduced by 15% due to friction along the vanes. If the discharge at outlet is without whirl find the ratio of bucket speed to the jet speed. If the jet diameter is 100 mm while the wheel diameter is 1.2 m, find the speed of the turbine in rpm, the force exerted by the jet on the wheel, the Power developed and the hydraulic efficiency. Take $C_v=0.97$.

**Solution:**

$H = 640 \text{ m}; \phi=15^\circ; V_{r1} = 0.85 \times V_{r2}; V_{w2} = 0; d = 100 \text{ mm}; D = 1.2 \text{ m}; C_v = 0.97; K_n = ?; N = ?; F_x = ?; P = ?; \eta_h = ?$

We know that the absolute velocity of jet is given by

\[
V = C_v \sqrt{2gH} = 0.97 \sqrt{2 \times 10 \times 640} = 109.74 \text{ m/s}
\]

Let the bucket speed be $u$

Relative velocity at inlet = $V_{r1} = V_f - u = 109.74 - u$

Relative velocity at outlet = $V_{r2} = (1-0.15)V_{r1} = 0.85(109.74 - u)$

But $V_{r2}\cos\phi = u \Rightarrow 0.85(109.74 - u)\cos15$

Hence $u = 49.48 \text{ m/s}$

But $u = \frac{\pi DN}{60}$ and hence
\[ N = \frac{60u}{\pi D} = \frac{60 \times 49.48}{\pi \times 1.2} = 787.5 \text{ rpm (Ans)} \]

Jet ratio \( m = \frac{u}{V} = \frac{49.48}{109.74} = 0.45 \)

Weight of water supplied \( \gamma Q = 10 \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^2 = 8.62 \text{ kN/s} \)

Force exerted \( F_x = \rho a V_1 (V_{w1} - V_{w2}) \)
But \( V_{w1} = V_1 \) and \( V_{w2} = 0 \) and hence
\[ F_x = 1000 \times \frac{\pi}{4} \times 0.1^2 (109.74)^2 = 94.58 \text{ kN} \]

Work done/second \( = F_x \times u = 94.58 \times 49.48 = 4679.82 \text{ kN/s} \)

Kinetic Energy/second \( = \frac{1}{2} \rho a V_1^3 = \frac{1}{2} \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^3 = 5189.85 \text{ kN/s} \)

Hydraulic Efficiency \( \eta_h = \frac{\text{Work done/s}}{\text{Kinetic Energy/s}} = \frac{4679.82}{5189.85} \times 100 = 90.17\% \)

Dec 06-Jan 07

A PELTON wheel turbine is having a mean runner diameter of 1.0 m and is running at 1000 rpm. The net head is 100.0 m. If the side clearance is 20° and discharge is 0.1 m³/s, find the power available at the nozzle and hydraulic efficiency of the turbine. (10)

**Solution:**

\( D = 1.0 \text{ m}; \ N = 1000 \text{ rpm}; \ H = 100.0 \text{ m}; \ \phi = 20°; \ Q = 0.1 \text{ m³/s}; \ WD/s = ? \) and \( \eta_h = ? \)

Assume \( C_v = 0.98 \)

We know that the velocity of the jet is given by
\[ V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 1000} = 43.83 \text{ m/s} \]

The absolute velocity of the vane is given by
\[ u = \frac{\pi D N}{60} = \frac{\pi \times 1 \times 1000}{60} = 52.36 \text{ m/s} \]

This situation is impracticable and hence the data has to be modified.

Clearly state the assumption as follows:
Assume \( H = 700 \) m (Because it is assumed that the typing and seeing error as 100 for 700)

Absolute velocity of the jet is given by
\[
V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 700} = 115.96 \text{ m/s}
\]

Power available at the nozzle is the given by work done per second
\[
\text{WD/second} = \gamma Q H = \rho g Q H = 1000 \times 10 \times 0.1 \times 700 = 700 \text{ kW}
\]

Hydraulic Efficiency is given by
\[
\eta_h = \frac{2u}{V_1} (V_1 - u) [1 + \cos \phi] = \frac{2 \times 52.36}{115.96^2} (115.96 - 52.36)(1 + \cos 20) = 96.07\%
\]

July 06
A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lps under a head of 30 m. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the coefficient of nozzle as 0.98.

Solution:
\[
u = 10 \text{ m/s}; \quad Q = 0.7 \text{ m}^3/\text{s}; \quad \phi = 180 - 160 = 20^\circ; \quad H = 30 \text{ m}; \quad C_v = 0.98;
\]

WD/s = ? and \( \eta_h = ? \)
Assume \( g = 10 \text{ m/s}^2 \)
\[ V = C_v \sqrt{gH} = 0.98 \sqrt{2 \times 10 \times 30} = 24 \text{ m/s} \]

\[ V_{r1} = V_I - u = 24 - 10 = 14 \text{ m/s} \]

Assuming no shock and frictional losses we have \( V_{r1} = V_{r2} = 14 \text{ m/s} \)

\[ V_{w2} = V_{r2} \cos \phi - u = 14 \times \cos 20 - 10 = 3.16 \text{ m/s} \]

We know that the Work done by the jet on the vane is given by

\[ \text{WD/s} = \rho \ a V_I \left[ V_{w1} + V_{w2} \right] u = \rho Q u \left[ V_{w1} + V_{w2} \right] \text{ as } Q = a V_I \]

\[ = 1000 \times 0.7 \times 10 \left[ 24 + 3.16 \right] = 190.12 \text{ kN-m/s (Ans)} \]

\[ \text{IP/s} = \text{KE/s} = \frac{1}{2} \rho a V_I^3 = \frac{1}{2} \rho Q V_I^2 = \frac{1}{2} \times 1000 \times 0.7 \times 24^2 = 201.6 \text{ kN-m/s} \]

Hydraulic Efficiency = Output/Input = \( \frac{190.12}{201.6} \approx 94.305\% \)

It can also be directly calculated by the derived equation as

\[ \eta_h = \frac{2u}{V_i^2} (V_i - u) \left[ 1 + \cos \phi \right] = \frac{2 \times 10}{24^2} (24 - 10) \left[ 1 + \cos 20 \right] = 94.29\% \text{ (Ans)} \]

Jan 06

A Pelton wheel has to develop 13230 kW under a net head of 800 m while running at a speed of 600 rpm. If the coefficient of Jet \( C_y = 0.97 \), speed ratio \( \phi = 0.46 \) and the ratio of the Jet diameter is \( 1 \div 16 \) of wheel diameter. Calculate

i) Pitch circle diameter
ii) the diameter of jet
iii) the quantity of water supplied to the wheel
iv) the number of Jets required.

Assume over all efficiency as 85%.

\textbf{Solution:}

\[ P = 13239 \text{ kW;} \quad H = 800 \text{ m;} \quad N = 600 \text{ rpm;} \quad C_v = 0.97; \quad \phi = 0.46 \text{ (Speed ratio)} \]

\[ \frac{d}{D} = 1/16; \quad \eta_o = 0.85; \quad D = ?; \quad d = ?; \quad n = ?; \]

Assume \( g = 10 \text{ m/s}^2 \) and \( \rho = 1000 \text{ kg/m}^3 \)

We know that the overall efficiency is given by

\[ \eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{\gamma Q H} = \frac{13239 \times 10^3}{10 \times 1000 \times 800} = 0.85 \]

Hence \( Q = 1.947 \text{ m}^3/\text{s} \) (Ans)

Absolute velocity of jet is given by

\[ V = C_v \sqrt{2 g H} = 0.97 \sqrt{2 \times 10 \times 800} = 122.696 \text{ m/s} \]

Absolute velocity of vane is given by

\[ u = \phi \sqrt{2 g H} = 0.46 \sqrt{2 \times 10 \times 800} = 58.186 \text{ m/s} \]

The absolute velocity of vane is also given by

\[ u = \frac{\pi D N}{60} \quad \text{and hence} \]

\[ D = \frac{60 u}{\pi N} = \frac{60 \times 58.186}{\pi \times 600} = 1.85 \text{ m} \] (Ans)

\[ d = \frac{1.85}{16} = 115.625 \text{ mm} \] (Ans)

Discharge per jet = \( q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times 0.115625^2 \times 122.696 = 1.288 \text{ m}^3/\text{s} \)

No. of jets = \( n = \frac{Q}{q} = \frac{1.947}{1.288} \approx 2 \) (Ans)

\[ \text{July 05} \]

Design a Pelton wheel for a head of 80m. and speed of 300 RPM. The Pelton wheel develops 110 kW. Take co-efficient of velocity = 0.98, speed ratio= 0.48 and overall efficiency = 80%.

\textbf{Solution:}

\[ H = 80 \text{ m;} \quad N = 300 \text{ rpm;} \quad P = 110 \text{ kW;} \quad C_v = 0.98, \quad K_u = 0.48; \quad \eta_o = 0.80 \]
Assume \( g = 10 \text{ m/s}^2 \) and \( \rho = 1000 \text{ kg/m}^3 \)

We know that the overall efficiency is given by

\[
\eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{\gamma Q H} = \frac{110 \times 10^3}{10 \times 1000 \times Q \times 80} = 0.8
\]

Hence \( Q = 0.171875 \text{ m}^3/\text{s} \)

Absolute velocity of jet is given by

\[
V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 80} = 39.2 \text{ m/s}
\]

Absolute velocity of vane is given by

\[
u = \phi \sqrt{2gH} = 0.48 \sqrt{2 \times 10 \times 80} = 19.2 \text{ m/s}
\]

The absolute velocity of vane is also given by

\[
u = \frac{\pi D N}{60} \text{ and hence}
\]

\[
D = \frac{60u}{\pi N} = \frac{60 \times 19.2}{\pi \times 300} = 1.22 \text{ m (Ans)}
\]

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation

\[
Q = \pi d^2 \times V = \pi \times d^2 \times 39.2 \Rightarrow 0.171875
\]

Hence \( d = 74.7 \text{ mm} \)

The design parameters are

Single jet

Pitch Diameter = 1.22 m

Jet diameter = 74.7 mm

Jet Ratio = \( m = \frac{D}{d} = \frac{1.22}{0.0747} = 16.32 \)

No. of Buckets = \( 0.5x + 15 = 24 \)

Jan 05

It is desired to generate 1000 kW of power and survey reveals that 450 m of static head and a minimum flow of 0.3 m\(^3\)/s are available. Comment whether the task can be accomplished by installing a Pelton wheel run at 1000 rpm and having an overall efficiency of 80\%.
Further, design the Pelton wheel assuming suitable data for coefficient of velocity and coefficient of drag.

**Solution:**

\( P = 1000 \text{ kW}; \ H = 450 \text{ m}; \ Q = 0.3 \text{ m}^3/\text{s}; \ N = 1000 \text{ rpm}; \ \eta_o = 0.8 \)

Assume \( C_v = 0.98; \ K_u = 0.45; \ \rho = 1000 \text{ kg/m}^3; \ g = 10 \text{ m/s}^2 \)

\[
\eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{\gamma Q H} = \frac{1000 \times 10^3}{10 \times 1000 \times 0.3 \times 450} = 0.74
\]

For the given conditions of \( P, Q \) and \( H \), it is not possible to achieve the desired efficiency of 80%.

To decide whether the task can be accomplished by a Pelton turbine compute the specific speed \( N_s \)

\[
N_s = \frac{N \sqrt{P}}{H^{5/4}};
\]

where \( N \) is the speed of runner, \( P \) is the power developed in kW and \( H \) is the head available at the inlet.

\[
N_s = \frac{1000 \sqrt{1000}}{450^{5/4}} = 15.25 < 35
\]

Hence the installation of single jet Pelton wheel is justified.

Absolute velocity of jet is given by

\[
V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 450} = 92.97 \text{ m/s}
\]

Absolute velocity of vane is given by

\[
u = \phi \sqrt{2gH} = 0.48 \sqrt{2 \times 10 \times 80} = 19.2 \text{ m/s}
\]

The absolute velocity of vane is also given by

\[
u = \frac{\pi D N}{60} \quad \text{and hence}
\]

\[
D = \frac{60 \nu}{\pi N} = \frac{60 \times 19.2}{\pi \times 300} = 1.22 \text{ m (Ans)}
\]

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation
$$Q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times d^2 \times 39.2 \Rightarrow 0.171875$$

Hence \( d = 74.7 \) mm

The design parameters are

Single jet
Pitch Diameter = 1.22 m
Jet diameter = 74.7 mm
Jet Ratio = \( m = \frac{D}{d} = \frac{1.22}{0.0747} = 16.32 \)

No. of Buckets = 0.5\( x \) + 15 = 24

July 04

A double jet Pelton wheel develops 895 MKW with an overall efficiency of 82% under a head of 60 m. The speed ratio = 0.46, jet ratio = 12 and the nozzle coefficient = 0.97. Find the jet diameter, wheel diameter and wheel speed in RPM.

**Solution:**

No. of jets = \( n = 2; \) \( P = 895 \) kW; \( \eta_o = 0.82; \) \( H = 60 \) m; \( K_n = 0.46; \) \( m = 12; \)
\( C_v = 0.97; \) \( D = ?; \) \( d = ?; \) \( N = ? \)

We know that the absolute velocity of jet is given by

\[ V = C_v \sqrt{2gH} = 0.97 \sqrt{2 \times 10 \times 60} = 33.6 \text{ m/s} \]

The absolute velocity of vane is given by

\[ u = K_v \sqrt{2gH} = 0.46 \sqrt{2 \times 10 \times 60} = 15.93 \text{ m/s} \]

Overall efficiency is given by

\[ \eta_o = \frac{P}{\gamma QH} \text{ and hence } Q = \frac{P}{\gamma \eta H} = \frac{895 \times 10^3}{10 \times 10^3 \times 0.82 \times 60} = 1.819 \text{ m}^3/\text{s} \]

Discharge per jet = \( q = \frac{Q}{n} = \frac{1.819}{2} = 0.9095 \text{ m}^3/\text{s} \)

From discharge continuity equation, discharge per jet is also given by

\[ q = \frac{\pi d^2}{4} \times V = \frac{\pi d^2}{4} \times 33.6 \Rightarrow 0.9095 \]

\( d = 0.186 m \)
Further, the jet ratio \( m = 12 = \frac{D}{d} \)

Hence \( D = 2.232 \) m

Also \( u = \frac{\pi D N}{60} \) and hence \( N = \frac{60u}{\pi D} = \frac{60 \times 15.93}{\pi \times 2.232} = 136 \) rpm

Note: Design a Pelton wheel: Width of bucket = 5\( d \) and depth of bucket is 1.2\( d \)

The following data is related to a Pelton wheel:

Head at the base of the nozzle = 80 m; Diameter of the jet = 100 mm;
Discharge of the nozzle = 0.3 \( m^3/\)s; Power at the shaft = 206 kW; Power absorbed in mechanical resistance = 4.5 kW. Determine (i) Power lost in the nozzle and (ii) Power lost due to hydraulic resistance in the runner.

Solution:

\( H = 80 \) m; \( d = 0.1 \) m; \( a = \frac{1}{4} \pi d^2 = 0.007854 \) m\(^2\); \( Q = 0.3 \) \( m^3/\)s; SP = 206 kW; Power absorbed in mechanical resistance = 4.5 kW.

From discharge continuity equation, we have,

\[ Q = a \times V = 0.007854 \times V \Rightarrow 0.3 \]

\[ V = 38.197 \) m/s

Power at the base of the nozzle = \( \rho g Q H \)

\[ = 1000 \times 10 \times 0.3 \times 80 = 240 \) kW

Power corresponding to the kinetic energy of the jet = \( \frac{1}{2} \rho a V^3 \)

\[ = 218.85 \) kW

(i) Power at the base of the nozzle = Power of the jet + Power lost in the nozzle

Power lost in the nozzle = 240 — 218.85 = 21.15 kW (Ans)

(ii) Power at the base of the nozzle = Power at the shaft + Power lost in the

(nozzle + runner + due to mechanical resistance)

Power lost in the runner = 240 — (206 + 21.15 + 4.5) = 5.35 kW (Ans)
The water available for a Pelton wheel is 4 m$^3$/s and the total head from reservoir to the nozzle is 250 m. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipeline is 3000 m long. The efficiency if power transmission through the pipeline and the nozzle is 91% and efficiency of each runner is 90%. The velocity coefficient of each nozzle is 0.975 and coefficient of friction $4f$ for the pipe is 0.0045. Determine:

(i) The power developed by the turbine; (ii) The diameter of the jet and (iii) The diameter of the pipeline.

**Solution:**

\[ Q = 4 \text{ m}^3/\text{s}; \quad H_g = 250 \text{ m}; \quad \text{No. of jets} = n = 2 \times 2 = 4; \quad \text{Length of pipe} = l = 3000 \text{ m}; \]

Efficiency of the pipeline and the nozzle $= 0.91$ and Efficiency of the runner $= \eta_h = 0.9$; $C_v = 0.975$; $4f = 0.0045$

Efficiency of power transmission through pipelines and nozzle $= \eta = \frac{H_g - h_f}{H_g}$

\[ \Rightarrow 0.91 = \frac{250 - h_f}{250} \]

Hence $h_f = 22.5$ m

Net head on the turbine $= H = H_g - h_f = 227.5$ m

Velocity of jet $= V_i = C_v \sqrt{\frac{2gH}{H_g}} = 0.975 \sqrt{2 \times 10 \times 227.5} = 65.77$ m/s

(i) Power at inlet of the turbine $= WP = $ Kinetic energy/second $= \frac{1}{2} \rho a V^3$

\[ WP = \frac{1}{2} \times 4 \times 65.77^2 = 8651.39 \text{ kW} \]

\[ \eta_h = \frac{\text{Power developed by turbine}}{WP} = \frac{8651.39}{WP} \Rightarrow 0.9 \]

Hence power developed by turbine $= 0.9 \times 8651.39 = 7786.25 \text{ kW}$ (Ans)

(ii) Discharge per jet $= q = \frac{\text{Total discharge}}{\text{No. of jets}} = \frac{4.0}{4} = 1.0 \text{ m}^3/\text{s}$

\[ \text{But } q = \frac{\pi}{4} d^2 \times V_i \Rightarrow 1.0 = \frac{\pi}{4} d^2 \times 65.77 \]

Diameter of jet $= d = 0.14$ m (Ans)

(iii) If $D$ is the diameter of the pipeline, then the head loss through the pipe is given by $= h_f$
$$h_f = \frac{4fLV^2}{2gD} = \frac{fLQ^2}{3D^5} \quad \text{(From } Q = aV)$$

$$h_f = \frac{0.0045 \times 3000 \times 4^2}{3D^5} \Rightarrow 22.5$$

Hence $D = 0.956 \text{ m (Ans)}$

The three jet Pelton wheel is required to generate 10,000 kW under a net head of 400 m. The blade at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ and the speed ratio = 0.46, then find: (i) the diameter of the jet, (ii) total flow (iii) the force exerted by a jet on the buckets (iv) The speed of the runner.

**Solution:**

No of jets = 3; Total Power $P = 10,000 \text{ kW}$; Net head $H = 400 \text{ m}$; Blade angle = $\phi = 15^\circ$; $Vr_2 = 0.95$ $Vr_1$; Overall efficiency = $\eta_o = 0.8$; $C_v = 0.98$; Speed ratio = $K_u = 0.45$; Frequency = $f = 50 \text{ Hz/s}$.

We know that $\eta_o = \frac{P}{\rho g QH} \Rightarrow 0.8 = \frac{10,000 \times 10^3}{1000 \times 10 \times Q \times 400}$

$Q = 3.125 \text{ m}^3/\text{s}$ (Ans)

Discharge through one nozzle = $q = \frac{Q}{n} = \frac{3.125}{3} = 1.042 \text{ m}^3/\text{s}$

Velocity of the jet = $V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 400} = 87.65 \text{ m/s}$

But $q = \frac{\pi}{4} d^2 \times V_1 \Rightarrow 1.042 = \frac{\pi}{4} d^2 \times 87.65$

$d = 123 \text{ mm (Ans)}$

Velocity of the Vane = $u = K_u \sqrt{2gH} = 0.46 \sqrt{2 \times 10 \times 400} = 41.14 \text{ m/s}$

$Vr_1 = (V_1 - u_1) = 87.65 - 41.14 = 46.51 \text{ m/s}$

$Vr_2 = 0.95$ $Vr_1 = 0.95 \times 46.51 = 44.18 \text{ m/s}$

$V_{w1} = V_1 = 87.65 \text{ m/s}$

$V_{w2} = Vr_2 \cos \phi - u_2 = 44.18 \cos 15 - 41.14 = 1.53 \text{ m/s}$

Force exerted by the jet on the buckets = $F_x = \rho q (V_{w1} + V_{w2})$
\[ F_x = 1000 \times 1.042 \times (87.65 + 1.53) = 92.926 \text{ kN (Ans)} \]

Jet ratio = \( m = \frac{D}{d} \Rightarrow 10 \) (Assumed)

\[ D = 1.23 \text{ m} \]

\[ u = \frac{\pi D N}{60} \]

Hence \( N = \frac{60u}{\pi D} = \frac{60 \times 41.14}{\pi \times 1.23} = 638.8 \text{ rpm (Ans)} \)